

I B. Tech II Semester Regular Examinations, December - 2020

MATHEMATICS-III

(Common to ALL Branches)

Time: 3 hours

Max. Marks: 60

Note : Answer **ONE** question from each unit (**5 × 12 = 60 Marks**)

UNIT - I

1. a) Find rank of $A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by reducing into Echelon form. 6M
- b) For what values of 'a' and 'b' the system of equations 6M
 $x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + az = b$ has
 i) No solution ii) Unique solution iii) Infinite number of solutions.

OR

2. a) Find the Eigen values and the corresponding Eigen vectors of the matrix 6M

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$
- b) Solve $5x + 10y + z = 28$; $4x + 8y + 3z = 29$; $x + y + z = 6$ by using Gauss 6M
 Jordan method

UNIT - II

3. a) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1} 6M
 and A^4 .
- b) Reduce the matrix $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ into diagonal matrix and find A^6 . 6M

OR

4. Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy + 4xz - 2yz$ to a canonical 12M
 form by orthogonal transformation method. Find Index, Rank, Signature and
 Nature of the quadratic form.

UNIT - III

5. a) Calculate the angle between the normal to the surface $xy - z^2 = 9$ at points 6M
 (4, 1, 2) and (3, 3, -3).
- b) Find the values of a and b so that the surfaces $ax^2 - byz = (a+2)x$ and 6M
 $4x^2y + z^3 = 4$ intersect orthogonally at (1, -1, 2).

OR

6. a) Find a, b, c such that $\vec{F} = (2x + 3y + az)\vec{i} + (bx + 2y + 3z)\vec{j} + (2x + cy + 3z)\vec{k}$ is irrotational. 6M
- b) Show that $\nabla^2 r^n = n(n+1)r^{n-2}$. 6M

UNIT – IV

7. Apply Green's theorem to evaluate $\oint_C (2xy - x^2)dx + (x^2 + y^2)dy$ where C is the region bounded by $x = y^2$ and $y = x^2$. 12M

OR

8. a) If $\vec{F} = (5xy - 6x^2)\vec{i} + (2y - 4x)\vec{j}$ and C is the curve $y = x^3$ in xy - plane. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ from (1, 1) to (2, 8). 6M
- b) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ where taken around the rectangle bounded by the lines $x = \pm a, y = 0, y = b$. 6M

UNIT - V

9. a) Form a partial differential equation by eliminating arbitrary function from the equation $z = xy + f(x^2 + y^2)$ 6M
- b) Solve $(yz)p + (zx)q = xy$ 6M

OR

10. a) Solve $z^2(p^2 + q^2 + 1) = 1$ 6M
- b) Solve $(D^2 - 4DD' + 4D'^2)z = 0$ 6M
